Neighborhood Sorting In and Out of Equilibrium

Gregorio Caetano and Vikram Maheshri*

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Abstract

We present an empirical framework to study segregation that bridges the empirical literature on residential choice and the theoretical literature on neighborhood segregation. The former literature features equilibrium empirical models of disaggregated choices, whereas the latter literature is concerned with the aggregate phenomenon of segregation, which is often studied in disequilibrium. With this in mind, our framework explicitly allows for the disaggregated households' choices to be observed out of equilibrium. We estimate the endogenous determinants of households' choices with novel instrumental variables that can be constructed with no additional data requirements. We then present a simulation procedure that aggregates these choices for a full characterization of the process of segregation. We implement our approach to analyze the racial segregation of White, Black, Hispanic and Asian households in the San Francisco Bay Area from 1990-2004. We find that households of all races react in very different ways to neighborhoods of different racial compositions. As a result of this heterogeneity, we find that neighborhoods at the end of our sample period are out of equilibrium, as segregation would increase by over 20% in the absence of any external shocks to the housing market.

*University of Rochester and University of Houston. We thank Patrick Bayer and Alvin Murphy for valuable conversations. All errors are our own. We also gratefully acknowledge Patrick Bayer and Alvin Murphy for sharing their data.

1 Introduction

Residential neighborhoods have been linked to a broad set of outcomes including educational attainment (Case and Katz (1991)), employment and wages (O'Regan and Quigley (1996)), access to food (Morland et al. (2002)), general health (Diez Roux (2001)) and social cohesion (Sampson, Raudenbush and Earls (1997)). Most of this literature has examined the effects of segregation in neighborhoods and schools along race and other socio-economic factors. For instance, residential segregation in neighborhoods have been linked to a broad set of outcomes including educational attainment and labor market outcomes (Cutler, Glaeser and Vigdor (2008)), infant health (Mason et al. (2009)), friendship formation (Mouw and Entwisle (2006)), crime (Kling, Ludwig and Katz (2005)), intergenerational mobility and economic opportunity (Chetty et al. (2014)) and various measures of subjective well being (Ludwig et al. (2012)). Similarly, school segregation has been linked to lower educational attainment (Rivkin, Hanushek and Kain (2005)) and wider black-white achievement gaps (Card and Rothstein (2007)) but has not been found to generate long lasting classroom peer effects (Angrist and Lang (2004)). In addition, school desegregation programs have been found to have increased black graduation rates (Guryan (2004)), college attendance and likelihood of arrest (Bergman (2016)).

Neighborhood and school segregation are aggregate outcomes that are determined by residential choices. Households with different preferences and expectations tend to choose to reside in different locations. If similar households are likely to have more similar preferences and expectations, we should observe an agglomeration of similar households in the same location, an outcome commonly known as segregation. Ever since Tiebout (1956), a rich theoretical and empirical literature has been developed to study residential choices. Specifically with respect to segregation, there has been two strands of the literature. First, empirical models of residential choice have focused on studying the determinants of segregation in equilibrium (e.g., Bayer, McMillan and Rueben (2004); Bayer and Timmins (2005, 2007); Ringo (2013)). Second, there have been advances in the understanding of segregation as an aggregate phenomenon in disequilibrium models of segregation based on the seminal work of Schelling (1969, 1971). In this paper, we present a unified framework that attempts to build a bridge between these two strands of the literature. We then apply our approach to analyze racial segregation in the San Francisco Bay Area from 1990 to 2004.

A key assumption in models of residential choice is that households are observed in *sorting equilibrium* (Bayer and Timmins (2005)), i.e., in the absence of future shocks, the demographic compositions of the neighborhoods will not change. This assumption is somewhat incongruous with the data. In Figure 2, we present the racial compositions of several neighborhoods in the San Francisco Bay Area over a fifteen year period. These neighborhoods exhibit substantial demographic changes, and these changes often appear to be serially correlated. To fit this prediction, standard empirical models of residential choice attribute these changes to (serially correlated) exogenous shocks to these neighborhoods.

However, as Schelling (1969) first argued, the composition of neighbors may also change endogenously due to the presence of neighborhood amenities that not only affect but are also affected by residential decisions. If, for instance, White households prefer White neighbors relative to non-White households, then an increase in the non-White share of a neighborhood could induce additional relative outflows of White households, which will endogenously contribute to further White flight even in the absence of exogenous shocks. The dynamics induced by such social interactions has led Schelling (1971) to suggest that neighborhoods are more likely to be observed adjusting along some trajectory to a long-run equilibrium rather than having achieved that equilibrium already.

Specifically, our paper has four main methodological innovations. (1) We develop a two-step approach to study neighborhood segregation along a social, endogenous amenity, such as the racial composition of neighbors. It allows for households to make residential decisions with incomplete information, thus allowing for neighborhoods to not converge to an equilibrium instantaneously. Thus, at any point in time neighborhoods may be observed out of equilibrium, i.e., in a trajectory towards convergence to an equilibrium. (2) This approach identifies the location and other features (e.g., stability) of any equilibrium, including in the presence of multiple equilibria. (3) We implement this approach in a dynamic model of residential choice with moving costs. Importantly, we show that for our purposes we can avoid making many of the assumptions made in the standard literature regarding the structure of state variables and their expected transitions, which allows us to circumvent the inherent issues involved with separately identifying preferences and expectations raised in Manski (2004).[1] This happens because our approach does not require isolating the flow utility component of a cumulative utility. (4) We introduce novel instrumental variables (IV) to identify the causal effects of endogenous, social amenities on demand. These IVs have no additional data requirements and follow directly from the internal logic of a dynamic model of residential choice with moving costs, even though they can be also used without imposing such assumptions, e.g., in a reduced-form context.

We implement our approach to analyze racial segregation between White, Black, Hispanic and Asian households in the San Francisco Bay Area from 1990-2004 using a recently constructed, high frequency data set on households' residential moves in the Bay Area (Bayer et al. (2016)). We find that households of different races react quite differently to neighborhoods of different racial compositions. Households of all races react positively to neighbors of the same race, though to differing degrees (e.g., Hispanic households react much more positively to Hispanic neighbors than households of other races do). However, White and Asian households react negatively to Black and Hispanic neighbors, and White, Black and Hispanic households all react modestly negatively to Asian neighbors.

These heterogeneous responses underly how households sort across neighborhoods, which, in the aggregate, leads to a distinct pattern of increasing segregation. We find that in the absence of external shocks, segregation in the Bay Area in 2004 would increase by over 20% relative to the current year. Households initially sort slowly, as the implied changes in the expected racial composition of neighborhoods are not sufficiently large to offset most households' moving costs. However, as these small changes accumulate, after two to four years over two thirds of Bay Area neighborhoods experience a substantial amount of monthly

[1] As Manski (2004) has pointed out, expectations and preferences may be undistinguishable using choice data alone. This may, for example, lead to misidentification of households' preferences for neighbors of the same race. For example, a world in which people care strongly for neighbors of the same race generates observationally equivalent choice data to a world in which people care weakly for neighborhoods of the same race, but tend to overestimate the proportion of neighbors of the same race. In our model, we can circumvent this issue by focusing on households' choices instead of their preferences since in both worlds, the racial compositions of neighborhoods would evolve identically.

turnover. The effects of this aggregate increase in segregation are not felt equally, as Hispanic households concentrate to the greatest extent, while White households actually spread themselves more diffusely throughout the neighborhoods of the Bay Area.

Relevant Literature

Our paper lies at the nexus of two distinct but related literatures related to neighborhood choice and segregation. We briefly review some of the most relevant studies.

Empirical Models of Residential Choice and Neighborhood Sorting

Because segregation is an outcome brought up by neighborhood sorting, a key related literature involves studying the residential choice of households. Ever since Tiebout (1956), these has been a prolific literature studying the determinants of residential choice, and thus segregation.[2] Three papers are particularly related to our study. Bayer, McMillan and Rueben (2004) develop a framework to estimate horizontal models of neighborhood choice, building on insights from the empirical Industrial Organization literature (Berry (1994) and Berry, Levinsohn and Pakes (1995)). This framework has been widely applied and extended in this literature (e.g., Bayer, Ferreira and McMillan (2007); Bayer, Keohane and Timmins (2009); Ringo (2013); Bayer et al. (2016); Caetano (2016)). They also discuss endogeneity concerns when a social, endogenous amenity is present, such as the composition of neighbors. Bayer and Timmins (2005) study the existence and uniqueness of equilibrium in such sorting models with social, endogenous amenities, and Bayer and Timmins (2007) discuss estimation in empirical models like these, suggesting an Instrumental Variables (IV) approach following the logic of a static model of neighborhood choice.

We build a new framework borrowing many insights from these papers. The

[2] See, for example, Epple, Filimon and Romer (1984); Kiel and Zabel (1996); Epple and Sieg (1999); Epple, Romer and Sieg (2001, 2003); Bayer, McMillan and Rueben (2004); Bayer and Timmins (2005, 2007); Bayer, Ferreira and McMillan (2007); Bayer et al. (2016); Caetano (2016). Kuminoff, Smith and Timmins (2013) provide a comprehensive review of the growing literature on neighborhood sorting.

key difference lies in our assumption regarding households' expectations at the time of their residential decision. While these models make assumptions of complete or nearly complete information, we build a framework that is agnostic about how expectations are formed. We discuss below that this departure is crucial to allow for the data to be observed out of the equilibrium. Another important difference is that our framework builds on a dynamic model of residential choice with moving cost. This is not new in the literature (see, for example, Bayer et al. (2016) and Caetano (2016)), but we show that, when the goal of the framework is to study segregation rather than to estimate preferences for amenities (which is the typical objective in these studies), then many assumptions made in standard approaches of dynamic demand estimation are not needed. Finally, we develop a novel IV approach that follows the logic of a *dynamic* model of neighborhood choice.

Models of Segregation based on Schelling (1969, 1971)

A largely theoretical literature based on the seminal Schelling model (Schelling (1969, 1971)), has sought to explain the mechanism behind segregation. In the Schelling model, heterogeneous agents select where to live by simple rules of thumb. Although this is a purely heuristic model that is not based on the explicit optimization of any objective, it has generated valuable insight into the fundamental social feature that generates segregation: agents of different races must react systematically differently to the racial composition of their neighbors. Schelling also made explicit the role of some social friction to ensure that neighborhoods gradually evolve toward an equilibrium state (e.g., myopia as in the Schelling model). A number of later theoretical papers have embedded this intuition into a more standard economic framework (e.g., Becker and Murphy (2000)), yielding important results. For instance, in a simple utility maximization framework, Pancs and Vriend (2007) show that segregation may arise even when agents do not possess discriminatory preferences in the aggregate, and Zhang (2009) has shown that the stable equilibria of the Schelling model are also stable when considered in the context of an evolutionary game. Recently, there has been many attempts to estimate these models of segregation both in a reduced-form and a structural context (e.g., Card, Mas and Rothstein (2008a); Banzhaf and Walsh (2013); Ringo

(2013); Caetano and Maheshri (2015)).

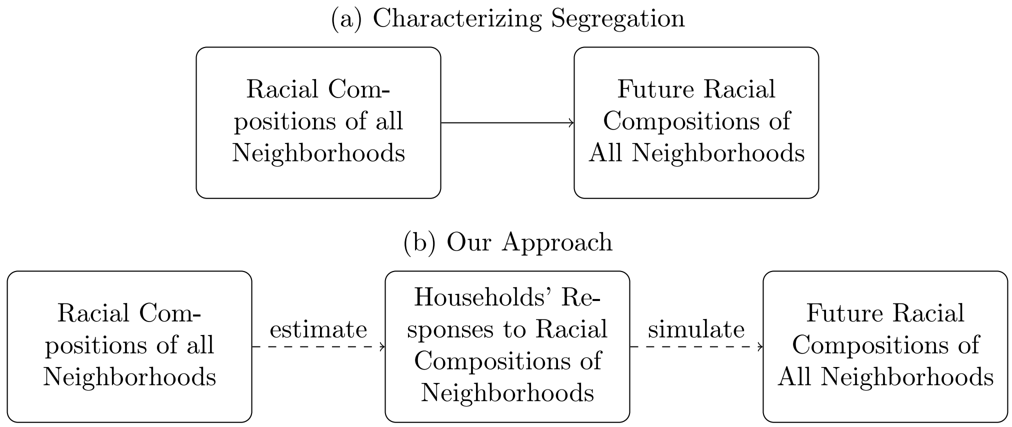
Like Caetano and Maheshri (2015), our paper provides an empirical framework that is both consistent with heuristic Schelling-style models and has an equivalent interpretation based on the optimization of a well defined objective function. We extend the ideas in Caetano and Maheshri (2015) in many directions. One key difference lies in the assumptions of expectations, which encompass not only the assumption made in models based on Schelling (1969, 1971), but also the assumptions made in standard models of neighborhood choice, as discussed above. Also, we focus in this paper on general equilibria, rather than on partial equilibria. Finally, we build our framework allowing for additional frictions in the choice model, such as moving costs, yielding novel IVs.

The rest of the paper proceeds as follows. In Section 2, we present an informal overview and a formal, if overly general, conceptualization of our empirical framework. In Section 3, we describe our data set, with the understanding that limitations in what we are able to observe as researchers must be acknowledged when implementing our approach. Accordingly, in Section 4, we present a detailed, structural implementation of our approach in the context of an estimable dynamic discrete choice model of residential choice and a companion simulation procedure. In Section 5, we present our empirical results before concluding in Section 6.

2 Empirical Approach

Before describing our empirical approach formally, we provide a big picture view of our objective in Figure 1. Fundamentally, studying segregation boils down to understanding how the racial compositions of neighborhoods will evolve over time. That is, characterizing segregation is equivalent to identifying the aggregate, neighborhood-level causal relationship depicted in panel 1a. Our approach,

Figure 1: Overview



which we depict in panel 1b, is to unpack that aggregate causal relationship into disaggregate, household-level causal responses to the racial compositions of neighborhoods. These responses may be mediated through preferences for neighbors of a given race, how households expectations are formed, or through preferences for other amenities that may vary with the racial compositions of neighborhoods. As such, they should be understood simply as demand responses *per se*, not discriminatory preferences. We identify these responses by estimating households' demand responses to neighbors, which can be conceptualized in a reduced-form demand framework or in a structural model of residential choice. We then aggregate these responses through a simple simulation procedure to obtain the future racial compositions of neighborhoods.

In this section, we provide a general conceptualization of our approach with a reduced-form specification of demand and minimal parametric restrictions. After presenting our data, we will describe in full detail how we implement our approach in practice with a structural, dynamic discrete choice model of neighborhood demand.

Formally, a city is divided into J neighborhoods, each of which are populated by households of four races: White, Black, Hispanic and Asian.[3] Let N_{jt}^r represent the number of households of race $r \in \{W, B, H, A\}$ who reside in neighborhood j in

[3] In our empirical application, we restrict our analysis to these four races for simplicity, as they constitute well over 95% of the households in our sample. In principle, groups could be defined at more or less aggregate levels and along alternative dimensions per the application.

period t. In each period, each neighborhood possesses a single, multidimensional endogenous amenity: the racial composition of their residents[4], which we denote with a vector of racial shares $\boldsymbol{s_{jt}} = \left(s_{jt}^B, s_{jt}^H, s_{jt}^A\right)'$ ($s_{jt}^r = \frac{N_{jt}^r}{\sum_{r'} N_{jt}^{r'}}$). The racial compositions of all neighborhoods in the city can be represented by the state matrix $\boldsymbol{s_t}$ whose jth column is $\boldsymbol{s_{jt}}$. (Hereafter, we refer to all vectors and matrices in bold type.) At the beginning of each period t, households form expectations of their value of residing in each neighborhood and then choose where to reside.

Following Figure 1, our approach consists of two steps: first, we estimate how households of each race will react to changes in their expectations of the racial compositions of neighborhoods. With these estimates, we can simulate the neighborhood-level demands for each race under counterfactual expected racial compositions of neighborhoods. This yields a full characterization of segregation dynamics for the city, as we can determine the convergent trajectory from any initial state to the relevant equilibrium. For instance, this allows us to identify how the racial compositions of all neighborhoods in the city, as observed, would evolve in the absence of external shocks. This also allows us to characterize any equilibria, including tipping points, bifurcations and stable equilibria.

2.1 Step 1: Estimation

We start with the log-demand function:

$$\log N_{jt}^r = f_j^r(\boldsymbol{s_t^{r,e}}; \boldsymbol{\alpha^r}) + \epsilon_{jt}^r \tag{1}$$

where $f_j^r(\cdot; \boldsymbol{\alpha^r})$ is a function unique to each neighborhood-race combination, and $\boldsymbol{s_{jt}^{r,e}} = \left(s_{jt}^{r,e,B}, s_{jt}^{r,e,H}, s_{jt}^{r,e,A}\right)'$ represents expectations of $\boldsymbol{s_t}$ from the perspective of households of race r. The parameter vector $\boldsymbol{\alpha^r}$ represents the marginal effects of $\boldsymbol{s_t^{r,e}}$ on demand (i.e., $\vec{\nabla} f^r$).

This specification of demand is quite general. For example, it allows for the expected endogenous amenities of neighborhood $k \neq j$ to affect demand for neighborhood j in a flexible way.

Our goal in the first stage is to consistently estimate $\boldsymbol{\alpha^r}$ for all $r = \{W, B, H, A\}$. Because we observe N_{jt}^r and $\boldsymbol{s_t}$, but we do not observe $\boldsymbol{s_t^{r,e}}$, it is infeasible to estimate equation (1). To circumvent this issue, we attempt to use the actual,

[4]In Remark 2, we discuss our approach in the presence of multiple endogenous amenities.

observed vector $\boldsymbol{s_t}$ as proxy for $\boldsymbol{s_t^{r,e}}$, which yields

$$\log N_{jt}^r = f_j^r(\boldsymbol{s_t};\boldsymbol{\alpha^r}) + \underbrace{\epsilon_{jt}^r + f_j^r(\boldsymbol{s_t^{r,e}};\boldsymbol{\alpha^r}) - f_j^r(\boldsymbol{s_t};\boldsymbol{\alpha^r})}_{\eta_{jt}^r} \tag{2}$$

Note that we have not yet made any assumptions on (a) how expectations are formed, (b) the function $f_j^r(\cdot;\boldsymbol{\alpha^r})$, or (c) the error term ϵ_{jt}^r. In particular, the composite error term in equation (2), η_{jt}^r, may be correlated to $\boldsymbol{s_t}$, which would lead to endogeneity. In Section 2, we impose restrictions on the function $f_j^r(\cdot;\boldsymbol{\alpha^r})$ that are in line with the literature on neighborhood choice, and we introduce a novel instrumental variables approach to identify $\boldsymbol{\alpha^r}$.

2.2 Step 2: Simulation

Next, we consider how the racial compositions of neighborhoods might evolve under different counterfactual values of $\boldsymbol{s_t^{r,e}}$, which we denote generically as $\tilde{\boldsymbol{s}}$. To do so, we must first calculate the counterfactual demand of households of race r for neighborhood j when $\boldsymbol{s_t^{r,e}} = \tilde{\boldsymbol{s}}$. Given estimates of $\hat{\boldsymbol{\alpha}}^{\boldsymbol{r}}$ from the first stage, this is equal to

$$\log N_{jt}^r(\tilde{\boldsymbol{s}}) = f_j^r(\tilde{\boldsymbol{s}};\hat{\boldsymbol{\alpha}}^{\boldsymbol{r}}). \tag{3}$$

Given $\log N_{jt}^r(\tilde{\boldsymbol{s}})$, we can calculate

$$s_{jt}^r(\tilde{\boldsymbol{s}}) = \frac{N_{jt}^r(\tilde{\boldsymbol{s}})}{\sum_{r'} N_{jt}^{r'}(\tilde{\boldsymbol{s}})} \tag{4}$$

and obtain the matrix $\boldsymbol{s_t}(\tilde{\boldsymbol{s}})$, whose jth element is $\big(s_{jt}^B(\tilde{\boldsymbol{s}}), s_{jt}^H(\tilde{\boldsymbol{s}}), s_{jt}^A(\tilde{\boldsymbol{s}})\big)$. Of course, this approach can be repeated for any counterfactual value of $\tilde{\boldsymbol{s}}$, which offers a way to identify the function $\boldsymbol{s_t}(\cdot)$ by simulation. An equilibrium in this context will correspond to values of $\tilde{\boldsymbol{s}}$ such that $\boldsymbol{s_t}(\tilde{\boldsymbol{s}}) = \tilde{\boldsymbol{s}}$.

Remark 1. Households can be thought of as players in a game of imperfect information where the action space is the set of possible neighborhoods from which they can choose to reside. In this vein, we estimate best response functions in the first stage, and we identify subgame perfect Bayesian Nash equilibria by simulation in the second stage.

Remark 2. *Additional Endogenous Amenities.* In this paper, we assume that the racial composition of neighborhoods is the only endogenous amenity of interest. We could in principle allow for households to consider other endogenous amenities (e.g., home prices, the incomes of neighbors, etc.) with the appropriate data. However, even if we were able to identify the causal demand responses of households to (their expectations of) these additional endogenous amenities, we would be unable to simulate the evolution of neighborhoods in a logically consistent manner without making additional assumptions on how households' expectations of these additional endogenous amenities would be affected by $\tilde{\boldsymbol{s}}$ (and in turn how they might later affect $\tilde{\boldsymbol{s}}$). Without explicit data on how expectations of all endogenous amenities are (jointly) formed, these assumptions are unwarranted (Manski (2004)). Thus, it is advisable to choose the set of endogenous amenities parsimoniously, with the understanding that one must focus only on the single *primitive* dimension along which households sort. For a study of racial segregation, this dimension is naturally the racial composition of neighborhoods. For a study of, say, gentrification, this dimension might instead be the average incomes of neighborhood residents.

Remark 3. *Expectations and Equilibrium.* A key difference between our approach and previous approaches to study segregation (e.g., Bayer, McMillan and Rueben (2004), Bayer and Timmins (2005), Bayer and Timmins (2007)) is that ours does not assume that data are observed in equilibrium. Here, we show that this has a close connection with an assumption on expectations. First, note that $\boldsymbol{s_t}(\tilde{\boldsymbol{s}} = \boldsymbol{s_t^e}) = \boldsymbol{s_t}$, since by definition actual, observed choices in the data are made when $\tilde{\boldsymbol{s}} = \boldsymbol{s}_t^{r,e}$. Thus, assuming $\boldsymbol{s_t^{r,e}} = \boldsymbol{s_t}$ (*full information*), as assumed in these papers, implies assuming data are observed in equilibrium, since $\boldsymbol{s_t}(\boldsymbol{s_t^e}) = \boldsymbol{s_t} = \boldsymbol{s_t^e}$. Similarly, assuming small deviations of full information (e.g., information set is the same across households i apart from some zero mean private information) will imply data to be observed in equilibrium as well. Therefore, it is crucial that we do not impose many restrictions on the expectations of households at the time they make their choices, otherwise we would not be able to test whether the data are observed in equilibrium. Our approach allows us to be agnostic as to the exact way expectations are formed as well as the content of households' information sets at the time of their decisions, which allows us to test whether data is observed in equilibrium. We find that households' choices are not observed in equilibrium.

3 Data

We perform our analysis on a monthly sample of all San Francisco Bay Area neighborhoods from 1990 to 2004. We define the San Francisco Bay Area as the six core counties (Alameda, Contra Costa, Marin, Santa Clara, San Francisco and San Mateo counties) that comprises the major cities of San Francisco, Oakland and San Jose and their surroundings, and we divide the sample region into 225 neighborhoods. Neighborhoods are defined by merging contiguous Census tracts until each resulting neighborhood contains approximately 10,000 households. Those neighborhoods with fewer than six annual home sales in our sample period are dropped.

For each neighborhood in each month, we compute estimates of their racial composition following the approach used by Bayer et al. (2016) to construct our sample.[5] Because high frequency data on the racial composition of neighborhoods is unavailable from standard sources (e.g., the Census) we must merge information from two main sources in order to construct these variables. The first of these sources is Dataquick Information Services, a national real estate data service. Dataquick provides a detailed listing of *all* real estate transactions in the Bay Area including buyers' and sellers' names, buyer's mortgage information, transaction prices and property locations. The second of these sources is a a dataset on mortgage applications published in accordance with the Home Mortgage Disclosure Act (HMDA) of 1975. Notably, HMDA data contains demographic information on mortgage applicants and the locations of properties that the applicants are buying. By linking these datasets on buyer's mortgage information and property locations, we can uniquely match approximately 70 of all sales. In doing so, we are

[5]We gratefully acknowledge Bayer et al. (2016) for sharing their raw data and programs to convert this raw data into a usable dataset for our analysis.

able to estimate how the demographics of neighborhoods change with each real estate transaction. Given neighborhood level estimates of the flows of households of different races, we are able to compute estimates of the actual demographic composition of each neighborhood by anchoring our estimates to the actual racial composition of each neighborhood from the 1990 US Census. [6]

There are three basic results of this procedure: for each month of our sample period in each Bay Area neighborhood and for each of four races – White, Black, Hispanic and Asian – we observe the total number of homeowners, the total numbers of inflows to that neighborhood, and the total numbers of outflows from that neighborhood. We summarize our data in Table 1. The majority of homeowners in the Bay Area are White, although there are sizable Asian and Hispanic populations as well. The high variance in the race specific populations reflects substantial cross-sectional heterogeneity in the racial composition of neighborhoods. This composition also changes over time in our sample as reflected in net monthly inflows (inflows minus outflows) on the order of approximately 0.5%.

We preview the observed evolution of racial compositions over time for selected neighborhoods in Figure 2. The West Richmond, West Emeryville and Lake Merced neighborhoods all featured considerable Black pluralities in 1990. Over our sample period, they evolved into Hispanic, White and Asian neighborhoods respectively. This was largely driven by large relative inflows of households of those races. Meanwhile, the Portola neighborhood transitioned from a White to an Asian neighborhood, and the Alum Rock neighborhood transitioned from a White to a Hispanic neighborhood. Large influxes of Hispanic households into the North Richmond neighborhood transformed it from a mixed White and Black neighborhood to a predominantly Hispanic neighborhood. The evolving racial composition of these representative neighborhoods reflects (1) households that sort systematically, (2) households that sort sometimes at a fairly brisk pace, and (3) heterogeneity in the initial racial compositions of neighborhoods that will undergo turnover starting in 1990, and (4) heterogeneity in the terminal racial compositions of neighborhoods that underwent turnover as of 2004 .

To summarize, this stylized evidence casts reasonable doubt upon the standard

[6] Bayer et al. (2016) report the results of multiple diagnostic tests that ensure the validity of this estimation procedure.

Figure 2: Racial Composition of Selected Neighborhoods Over Time, 1990-2004 (1 of 2)

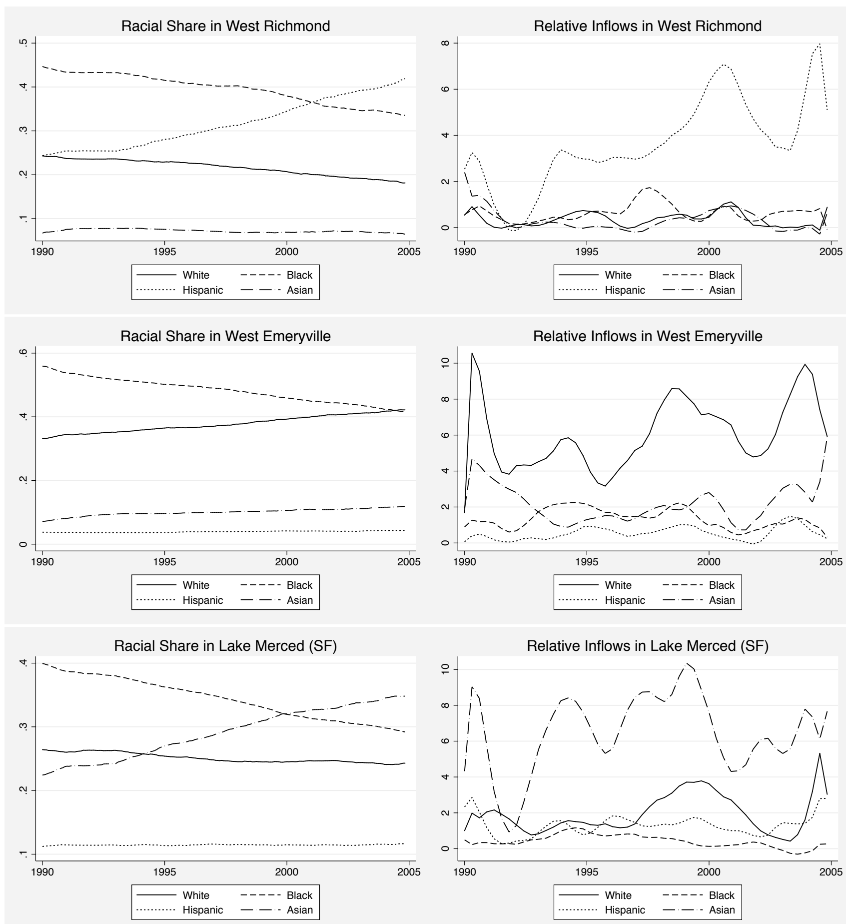


Figure 2: Racial Composition of Selected Neighborhoods Over Time, 1990-2004 (2 of 2)

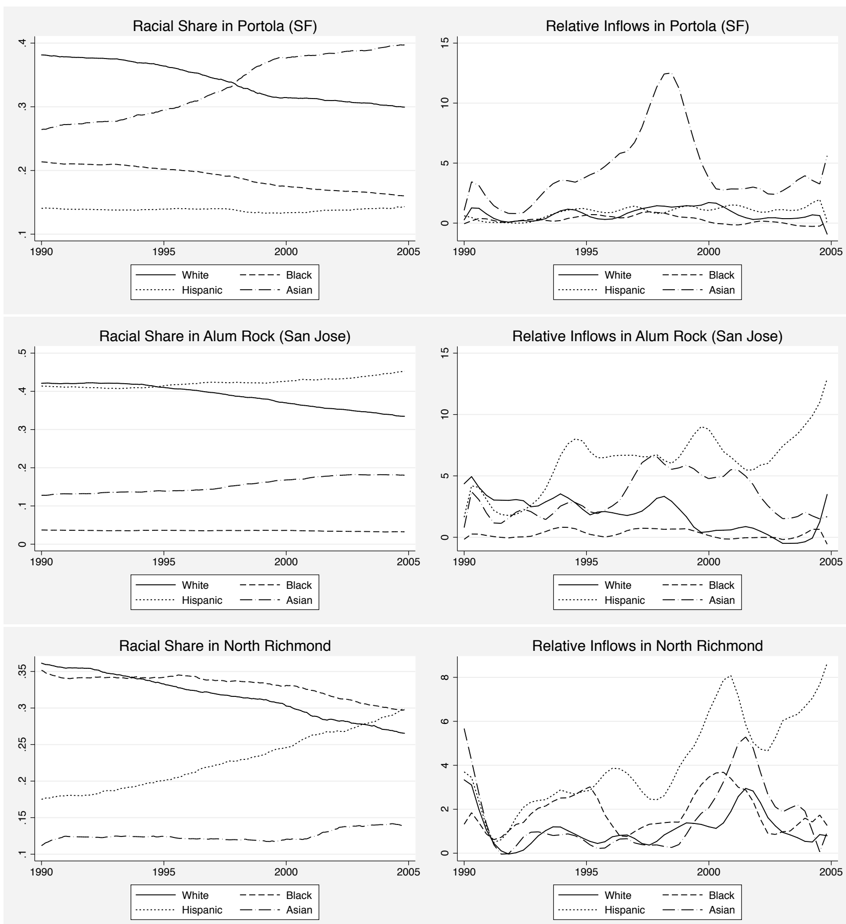


Table 1: Summary Statistics

Variable	White	Black	Hispanic	Asian
Average Number of Households per Neighborhood (N^r_{jt})	4100 (3073)	283 (456)	517 (577)	839 (1066)
Average Monthly Neighborhood Inflows ($N^{r,\text{ move}}_{jt}$)	11.08 (12.65)	0.68 (1.37)	2.20 (3.83)	4.74 (7.93)
Average Monthly Neighborhood Outflows	3.54 (6.05)	0.19 (0.60)	0.58 (1.56)	1.23 (3.09)
Average Home Sales Price (Thousands of 2004 Dollars[7])		390.48 (251.52)		
Total Number of Observations		40320		

Note: Standard deviations are presented in parentheses.

assumption that neighborhoods are observed in equilibrium.

4 Empirical Implementation

The empirical implementation of our approach must accommodate practical restrictions to the data that we can actually observe. Although the function $f^r_j(\cdot;\boldsymbol{\alpha^r})$ in principle allows for endogenous amenities in all neighborhoods to affect household demands in all other neighborhoods differentially, it would be infeasible to estimate such an expansive set of parameters $\boldsymbol{\alpha^r}$ without data of a much higher degree of detail than is available. Here, we impose some key restrictions that allow for the feasible estimation of this function. These restrictions derive from a specific dynamic discrete choice model that entails assumptions on the way in which the amenities of a neighborhood k may impact the demand of neighborhood j. We start by formally describing this model, and then we explain how we estimate it in three stages.

A Dynamic Model of Residential Choice

At the beginning of month t, households first choose whether or not to stay in the same house in which they lived in $t-1$. Conditional on deciding not to stay, they decide between one of the options $j = 1, ..., J+1$. Options $j = 1, ..., J$ correspond to owning a house in neighborhood j. Option $j = J+1$ corresponds to the outside option of either residing outside of the city or renting within the city (as in Bayer et al. (2016) we only observe data on homeowners). Households face a race and time varying moving cost equal to ϕ_t^r. To simplify notation, as in Bayer et al. (2016) we index the option of staying in the same house as $J+2$.

Let j_{it}^r represent the optimal choice of household i of race r in month t. For each j, r and t, we observe n_{jt}^r, the total number of households of race r who choose option $j_{it}^r = j$. For $j = 1, ..., J+1$, n_{jt}^r represents the number of inflows into neighborhood j from $t-1$ to t, and n_{J+2t}^r represents the number of "stayers", or households who decided to remain in the same house from $t-1$ to t.

We define the mean cumulative utility that households of race r obtain from owning a house in neighborhood j in month t as $v_{jt}^r\left(\epsilon_{ijt}^r\right)$. The ϵ_{ijt}^r contains household specific determinants of utility and is unobserved to the researcher. We can collect these choice-specific cumulative utilities into the vector $\boldsymbol{v_t^r}\left(\boldsymbol{\epsilon}_{it}^r\right)$ that contains $v_{jt}^r\left(\epsilon_{ijt}^r\right)$ as its jth element.

In each month, household i of race r observes the vector $(\boldsymbol{v_t^r}, \phi_t^r, j_{it-1}^r, \boldsymbol{\epsilon_{it}^r})$ and chooses j in order to maximize their cumulative utility given by

$$V_{ijt}^r(\boldsymbol{v_t^r}, \phi_t^r, j_{it-1}^r, \boldsymbol{\epsilon_{it}^r}) = \mathbb{I}_{\{j=J+2\}} \cdot \left(v_{j_{it-1}^r t}^r + \epsilon_{iJ+2t}^r\right) + \mathbb{I}_{\{j\in 1,...,J+1\}} \cdot \max_{k\in\{1,...,J+1\}} \left(v_{kt}^r - \phi_{j_{it-1}^r t}^r + \epsilon_{ikt}^r\right), \tag{5}$$

where $\mathbb{I}$ is the indicator function. The error term ϵ_{jt}^r is assumed to be i.i.d. extreme value 1.

Our approach unfolds in three stages: we first estimate v_{jt}^r and ϕ_t^r, and then we identify the causal effect of the expected endogenous amenity on v_{jt}^r, $\boldsymbol{\hat{\alpha}^r}$. Next, we use to these estimates to simulate the evolution of the racial compositions of neighborhoods from any given counterfactual value of the expected endogenous amenity.

Stage 1: Estimating Cumulative Household Utilities

First, we consider households that move, i.e., households for which $j_{it}^r \in \{1, \ldots J+1\}$. The decisions of these households can be used to identify the cumulative utilities v_{jt}^r. Conditional on moving, they make their choice by solving the following optimization problem:

$$\max_{k \in \{1,\ldots,J+1\}} v_{kt}^r - \phi_{j_{it-1}^r t}^r + \epsilon_{ikt}^r. \tag{6}$$

Based on the familiar logit formula, we can write choice-specific probabilities as:

$$\begin{aligned} P\left(j_{it}^r = j \,|\, j \in \{1.\ldots J+1\}, j_{it-1}\right) &= \frac{\exp(v_{jt}^r - \phi_{j_{it-1}t}^r)}{\sum_{k=1}^{J+1} \exp(v_{kt}^r - \phi_{j_{it-1}t}^r)} \\ &= \frac{\exp(v_{jt}^r)}{\sum_{k=1}^{J+1} \exp(v_{kt}^r)}. \end{aligned} \tag{7}$$

Because moving costs do not vary by option, they cancel out (conditional on moving).[8]

The data analog to the choice-specific probability is simply $\frac{n_{jt}^r}{\sum_{k=1}^{J+1} n_{kt}^r}$. Following Bayer et al. (2016) we can estimate $\hat{v}_{jt}^r$ for $j \in \{1, \ldots J\}$ as

$$\hat{v}_{jt}^r = \log\left(n_{jt}^r\right) - \log\left(n_{J+1t}^r\right) \tag{8}$$

where $\hat{v}_{J+1t}^r$ is normalized to zero (see, e.g., Berry (1994) and Berry, Levinsohn and Pakes (1995)).

Next, we consider households that stay in their house from $t-1$ to t. The decisions of these households, along with estimates of $\hat{v}_{jt}^r$, can be used to identify the moving cost parameters ϕ_{jt}^r. Because the probability that a household stays in their house should vary by the neighborhood in which they resided in $t-1$, we write the choice-specific probability for this option $(J+2)$ as:

[8] This insight is due to Bayer et al. (2016).

$$P\left(j_{it}^r = J+2 \mid j_{it-1}\right) = P\left(v_{jt}^r + \epsilon_{iJ+2t}^r > v_{kt}^r - \phi_{j_{it-1}t}^r + \epsilon_{ikt}^r \; \forall k \in \{1,...,J+1\} \mid j_{it-1}^r\right)$$
$$= \frac{\exp(v_{jt}^r)}{\exp(v_{jt}^r) + \sum_{k=1}^{J+1} \exp(v_{kt}^r - \phi_{j_{it-1}^r t}^r)} \tag{9}$$

where the last line follows from the standard logit formula. The data analog to $P\left(j_{it}^r = J+2 \mid j_{it-1} = j\right)$ is simply $\frac{stayers_{jt}^r}{N_{jt-1}^r}$ for $j = 1,...,J$. Hence, equation (9) suggests the moment restriction:

$$g_j(\phi_t^r; \boldsymbol{v_t^r}) = \frac{stayers_{jt}^r}{N_{jt-1}^r} - \sum_{j=1}^{J} \frac{\exp(v_{jt}^r)}{\exp(v_{jt}^r) + \sum_{k=1}^{J} \exp(v_{kt}^r - \phi_{jt}^r) + \exp(-\phi_{jt}^r)}, \quad j = 1,...,J \tag{10}$$

By substituting our estimates of $\hat{v}_{jt}^r$ from equation (8) into the moment condition for v_{jt}^r, we can estimate moving costs, ϕ_{jt}^r, via GMM. Note that we do not observe $stayers_{J+1t}^r$, so we do not identify ϕ_{J+1t}^r. We show below that this parameter estimate is not needed in our approach.

Stage 2: Estimating the Causal Effect of Endogenous Amenities on the Choice of Neighborhood

We can decompose the average cumulative utility that households of race r obtain from living in neighborhood j in month t as

$$v_{jt}^r = \boldsymbol{\alpha^{r\prime}} \boldsymbol{s_{jt}^{r,e}} + \gamma_t^r + \tilde{\xi}_{jt}^r, \quad j = 1,...,J \tag{11}$$

The parameters of interest, $\boldsymbol{\alpha^r}$, represent the causal effects of $\boldsymbol{s_{jt}^{r,e}}$ on v_{jt}^r, γ_t^r is a race-month fixed effect, and $\tilde{\xi}_{jt}^r$ is an error term that includes all remaining unobserved determinants of v_{jt}^r.

Because we do not observe v_{jt}^r or $\boldsymbol{s_{jt}^{r,e}}$, we use $\hat{v}_{jt}^r$ and $\boldsymbol{s_{jt}}$ as proxies for them. Accordingly, we rewrite equation (11) as

$$\hat{v}^r_{jt} = \boldsymbol{\alpha^{r\prime}} \boldsymbol{s_{jt}} + \gamma^r_t + \xi^r_{jt} \tag{12}$$

where the error term ξ^r_{jt} can be decomposed as

$$\xi^r_{jt} = \tilde{\xi}^r_{jt} + \boldsymbol{\alpha}^{r\prime}(\boldsymbol{s^{r,e}_{jt}} - \boldsymbol{s_{jt}}) + (\hat{v}^r_{jt} - v^r_{jt}) \tag{13}$$

The first term of equation (13) corresponds to unobserved determinants of households' cumulative utilities. The second term corresponds to errors in households' expectations. The third term corresponds to any bias in the estimation of households' cumulative utilities that was brought up from the first stage. This error term is likely correlated to $\boldsymbol{s_{jt}}$, leading to a biased OLS estimator of $\boldsymbol{\alpha^r}$. We address this endogeneity problem with synthetic instrumental variables that can be constructed from the logic of the neighborhood choice model.

Instrumental Variables

To identify $\boldsymbol{\alpha^r}$, we exploit the idea that $\hat{v}^r_{jt}$ is a *flow* variable, while $\boldsymbol{s_{jt}}$ is a *stock* variable. While $\hat{v}^r_{jt}$ reflects the value of neighborhood j as of month t, $\boldsymbol{s_{jt}}$ also reflects how the neighborhood was valued in prior months $t-1$, $t-2$, $\ldots$. We leverage this asymmetry to construct an IV that plausibly affects $\hat{v}^r_{jt}$ only through $\boldsymbol{s_{jt}}$. To do so, we isolate the transitory components of past valuations of neighborhood j. That is, we isolate the transitory shocks that affected $\boldsymbol{s_{jt}}$ through choices that households made in previous months, but no longer affect choices in month t.

Specifically, we use $\boldsymbol{s_{jt-2}}$ as an IV for $\boldsymbol{s_{jt}}$ in the following equation

$$\hat{v}^r_{jt} = \boldsymbol{\alpha^{r\prime}} \boldsymbol{s_{jt}} + \gamma^r_t + h^r(v^W_{jt-1}, v^B_{jt-1}, v^H_{jt-1}, v^A_{jt-1}) + \mu^r_{jt} \tag{14}$$

where $\mu^r_{jt} = \xi^r_{jt} - h^r(\cdot)$, and $h^r(v^W_{jt-1}, v^B_{jt-1}, v^H_{jt-1}, v^A_{jt-1})$ is a flexible control function.

Following the dynamic model of residential choice described above, s^r_{jt} and s^r_{jt-2} will be correlated because of shocks in $v^{r\prime}_{kt-2}, v^{r\prime}_{kt-3}, \ldots$ that either (a) persist until t or (b) do not persist until t but nevertheless affected households who moved to neighborhood j on or before $t-2$ and chose to remain until t due to moving costs. Shocks of type (a) will be problematic, as they may be correlated to the error term in equation (12), but shocks of type (b) will, by construction, be uncorrelated to that error term. By holding $\boldsymbol{v_{jt-1}}$ constant through the use of the control function in equation (14), we absorb the variation in our instruments that is due to shocks of type (a), thus isolating variation from shocks of type (b) alone. Put another way, our identifying assumption is that shocks to households' valuations of neighborhood j are Markov(1), i.e., shocks in $t-2$, $t-3$,... are uncorrelated to shocks in t conditional on shocks in $t-1$.

The logic underlying our exclusion restriction coupled with our relatively long panel of data can be leveraged to weaken the identifying assumption even further. For any given values of $T' > T$, we can use the component of $\boldsymbol{s_{jt-T'}}$ that is orthogonal to $(\boldsymbol{v_{jt-1}, ..., v_{jt-T}})$ as an IV for $\boldsymbol{s_{jt}}$. The corresponding identifying assumption is that shocks to households' valuations of neighborhood j are Markov(τ), i.e., shocks in $t-(T')$, $t-(T'+1)$,... are uncorrelated to shocks in t conditional on shocks in $t-1$,...,$t-\tau$, for $\tau \leq T$.[9] Thus, with larger choices of T and T', our identifying assumption will be further weakened.

Appendix A provides a more formal argument of why this IV follows logically from the dynamic choice model described above.[10]

Stage 3: Identifying Neighborhood Sorting Equilibria by Simulation

In any month, we can describe the entire city with race-specific population vectors $\boldsymbol{N_t^r} = (N_{1t}^r, \ldots, N_{Jt}^r)$ that imply racial specific share vectors $\boldsymbol{s_t^r} = (s_{1t}^r, \ldots s_{Jt}^r)$ and a racial composition matrix $\boldsymbol{s_t} = \left(\boldsymbol{s_t^{B\prime}, s_t^{H\prime}, s_t^{A\prime}}\right)^{\boldsymbol{\prime}}$.

As described in Figure (1a), the characterization of sorting and segregation is

[9] Because we use the cumulative utilities rather than the flow utilities as controls, our validity assumption is actually weaker than it seems. To see this, consider the simple example when $T=1$ and $T'=2$. The component of $\boldsymbol{s_{jt-2}}$ that is used as identifying variation can in principle be uncorrelated to v_{jt-1} but correlated to v_{jt}^r. However, the predictable component of v_{jt}^r (as of $t-1$) is already included in $\boldsymbol{v_{jt-1}}$ (because households are forward looking agents). Thus, the only instance in which our validity assumption will be violated is if a component of $\boldsymbol{s_{jt-2}}$ happens to show up again in v_{jt}^r *as a surprise to all races* (i.e., households of all races as of $t-1$ were not able to predict that shock in t).

[10] Although this identification strategy is motivated by a structural model, it can also be applied in a reduced-form demand estimation framework. Specifically, $\boldsymbol{s_{jt-T'}}$ can be used as instruments for $\boldsymbol{s_{jt}}$ conditional on the inflows of all races to neighborhood j between $t-1$ and $t-T$.

the understanding of how $\boldsymbol{s_t}$ evolves from any given state. To study this process, we choose counterfactual state vectors and simulate the co-evolution of household choices holding all other neighborhood characteristics fixed. Equation (12) describes how the expected racial composition in each neighborhood affect $\hat{v}^r_{jt}$ for each r and j. For any given counterfactual expected racial composition matrix $\tilde{\boldsymbol{s}} = (\tilde{\boldsymbol{s}}_{\boldsymbol{1}}, ..., \tilde{\boldsymbol{s}}_{\boldsymbol{J}})$ in period t, we can write the implied expected valuation for neighborhood j of race r households as

$$v^r_{jt}(\tilde{\boldsymbol{s}}) = \hat{\hat{v}}^r_{jt} + \hat{\boldsymbol{\alpha}}^{\boldsymbol{r}\prime}(\tilde{\boldsymbol{s}}_j - \boldsymbol{s_{jt}}) \tag{15}$$

where $\hat{\boldsymbol{\alpha}}^{\boldsymbol{r}}$ and $\hat{\hat{v}}^r_{jt}$, the predicted value of regression (14), are both estimated in the second stage. We can compute the implied race-specific demand for neighborhood $j = 1, ..., J$ as

$$\begin{aligned} N^r_{jt}(\tilde{\boldsymbol{s}}) \;=\;& N^r_{jt-1} \cdot \left(\frac{\exp\left(v^r_{jt}(\tilde{\boldsymbol{s}})\right)}{\exp\left(v^r_{jt}(\tilde{\boldsymbol{s}})\right) + \sum_{k'=1}^{J} \exp\left(v^r_{k't}(\tilde{\boldsymbol{s}}) - \hat{\phi}^r_{jt}\right) + \exp\left(-\hat{\phi}^r_{jt}\right)} \right) + \\ & + \sum_{k=1}^{J} N^r_{kt-1} \cdot \left(\frac{\exp\left(v^r_{jt}(\tilde{\boldsymbol{s}}) - \hat{\phi}^r_{kt}\right)}{\exp\left(v^r_{kt}(\tilde{\boldsymbol{s}})\right) + \sum_{k'=1}^{J} \exp\left(v^r_{k't}(\tilde{\boldsymbol{s}}) - \hat{\phi}^r_{kt}\right) + \exp\left(-\hat{\phi}^r_{kt}\right)} \right) \qquad (16) \\ & + inflows^r_t \cdot \left(\frac{\exp\left(v^r_{jt}(\tilde{\boldsymbol{s}})\right)}{\sum_{k'=1}^{J} \exp\left(v^r_{k't}(\tilde{\boldsymbol{s}})\right)} \right) \qquad (17) \end{aligned}$$

where $\hat{\phi}^r_t$ is estimated in the first stage, and $inflows^r_t$ represents the total inflows of households of race r from $t-1$ to t from the outside option to one of the "inside" options $j = 1, ..., J$. The first term on the right-hand side of equation (16) corresponds to the simulated number of households who resided in neighborhood j in $t-1$ and remain in their house, incurring no moving costs. The second term represents the simulated number of households who lived in neighborhood k in $t-1$ and then moved to neighborhood j ($k = j$ reflects those who move within neighborhood j). Because our simulation implicitly holds fixed all factors that affect households' propensity to choose the outside option, we only consider households who chose neighborhoods $j \in 1, ..., J$ in $t-1$ [11].

The implied demand in equation (16) is understood to be determined by a counterfactual manipulation of households' expectations (i.e., we set $\boldsymbol{s_t^{r,e}} = \tilde{\boldsymbol{s}}$) just before households make their decisions in t.[12] Together, equations (15) and (16) form a mapping from the neighborhood state space to itself. We can specify this with the implied racial composition function

$$s_{jt}^{r}\left(\tilde{\boldsymbol{s}}\right) = \frac{N_{jt}^{r}(\tilde{\boldsymbol{s}})}{\sum\limits_{r' \in R} N_{jt}^{r'}(\tilde{\boldsymbol{s}})} \tag{18}$$

which we collect into a well defined matrix valued function $\boldsymbol{s_t}(\tilde{\boldsymbol{s}}) : [0,1]^{3\times J} \rightarrow [0,1]^{3\times J}$ whose (r, j) element is equal to $s_{jt}^{r}\left(\tilde{\boldsymbol{s}}\right)$.

A sorting equilibrium (Bayer and Timmins (2005)) is defined as a state $\boldsymbol{s}^{\star}$ where in the absence of any shock ($\boldsymbol{\epsilon}$ or γ_t^r), there are no changes in the demographic compositions of the neighborhoods next period.

Definition 1. *Sorting Equilibria.* State $\boldsymbol{s}^{\star}$ is a *sorting equilibrium* if $\boldsymbol{s_t}(\boldsymbol{s}^{\star}) = \boldsymbol{s}^{\star}$.

In principle, we can identify all sorting equilibria as fixed points of the function $\boldsymbol{s_t}(\cdot)$ by conducting a grid search of all possible states $\tilde{\boldsymbol{s}}$ and computing $\boldsymbol{s_t}\left(\tilde{\boldsymbol{s}}\right)$ for each counterfactual. Given a sufficiently fine grid and tolerance δ, those states $\tilde{\boldsymbol{s}}$ for which $\|\boldsymbol{s_t}\left(\tilde{\boldsymbol{s}}\right) - \tilde{\boldsymbol{s}}\| < \delta$ can be interpreted as sorting equilibria. Because the domain of the grid search is too large ($[0,1]^{3\times J}$), it is computationally infeasible to identify all such equilibria. Thus, we restrict our attention to constructing the *simulated trajectory* $\left(\hat{\boldsymbol{s}}_{\boldsymbol{1}}, \hat{\boldsymbol{s}}_{\boldsymbol{2}} \ldots\right)$ where $\hat{\boldsymbol{s}}_{\boldsymbol{n}} = \boldsymbol{s_t}\left(\hat{\boldsymbol{s}}_{\boldsymbol{n-1}}\right)$ from a given initial state

[11] This is crucial to our analysis, as it allows us to use fixed point techniques to find equilibria even under a potentially non-stationary environment. For instance, since γ_t^r can vary over t and across r, we allow for race-specific secular changes to the attractiveness of the home-ownership market when estimating the parameters of the choice model. This is particularly important for an analysis of the Bay Area housing market during a period of rapid price appreciation.

[12] In principle, when choosing different counterfactuals, we could allow these expectations to vary by race: $\boldsymbol{s_t^{r,e}} \neq \boldsymbol{s_t^{r',e}}$. However, the dimensionality of the counterfactual – and hence the computational complexity of the simulation procedure – would increase by a factor of 4. Thus, in our implementation of this approach, we consider only counterfactuals where different races share the same expectations. This follows from the fact that at any equilibrium, $\boldsymbol{s_t^{r,e}} = \boldsymbol{s_t^{r',e}}$ for all r, r' must occur. Nonetheless, this restriction does constrain the deviations from equilibria that we can analyze in our simulation. Although such analysis is outside of the scope of the paper, it can be easily implemented with the approach developed here.

$\hat{\boldsymbol{s}}_{\boldsymbol{0}}$. For instance, an initial state we consider is $\hat{\boldsymbol{s}}_{\boldsymbol{0}} = \boldsymbol{s_t}$, as observed in the data. The state at which this trajectory converges corresponds to the particular sorting equilibrium to which the observed neighborhoods are predicted to converge (in the absence of future shocks). A comparison of this sorting equilibrium and $\boldsymbol{s_t}$ provides an explicit test of the assumption that data are observed in equilibrium.

This approach of simulating the trajectory given an initial state $\hat{\boldsymbol{s}}_{\boldsymbol{0}}$ is also useful to identify whether an equilibrium is stable or unstable.

Definition 2. *Stable and Unstable Equilibria.* Let $\|\cdot\|$ represent the $3 \times J$ dimensional Euclidean norm.

1. Equilibrium $\boldsymbol{s}^\star$ is *stable* if for any $\eta > 0$, there exists a δ and a n^δ such that $\|\boldsymbol{s_t}(\hat{\boldsymbol{s}}_{\boldsymbol{n}}) - \boldsymbol{s}^\star\| < \eta$ for all $\hat{\boldsymbol{s}}_0$ such that $\|\hat{\boldsymbol{s}}_{\boldsymbol{0}} - \boldsymbol{s}^\star\| < \delta$ and all $n > n^\delta$, where $\hat{\boldsymbol{s}}_{\boldsymbol{n}} = \boldsymbol{s_t}(\hat{\boldsymbol{s}}_{\boldsymbol{n-1}})$.

2. Equilibrium $\boldsymbol{s}^\star$ is unstable (i.e., a *tipping point*) if it is not stable.

Remark 4. It is worth discussing in detail why our IV is plausibly uncorrelated to $\boldsymbol{\alpha}^{r'}(\boldsymbol{s}^{\boldsymbol{r,e}}_{\boldsymbol{jt}} - \boldsymbol{s_{jt}})$, the second term included in ξ^r_{jt} in equation (13). Note that v^r_{jt} actually represents $v^{e,r}_{jt}$, the expected value of neighborhood j as of t, from the perspective of households of race r. Because $v^{e,r}_{jt}$ and $\boldsymbol{s}^{\boldsymbol{e,r}}_{\boldsymbol{jt}}$ reflect decisions made at the exact same time, it is plausible to assume that these expectations were formed with the same information. Thus, any component of $\boldsymbol{s_{jt}}$ affecting $v^{e,r}_{jt}$ must also affect $\boldsymbol{s}^{\boldsymbol{e,r}}_{\boldsymbol{jt}}$, or, more precisely, $\boldsymbol{s_{jt}}$ is excluded from equation $v^r_{jt} = \boldsymbol{\alpha^r}\boldsymbol{s}^{\boldsymbol{e,r}}_{\boldsymbol{jt}} + \gamma^r_t + f^r(\boldsymbol{v_{t-1}}) + \mu^r_{jt}$. It follows that any IV of $\boldsymbol{s_{jt}}$ would affect $v^{e,r}_{jt}$ only through $\boldsymbol{s}^{\boldsymbol{r,e}}_{\boldsymbol{jt}}$ and not through $\boldsymbol{s_{jt}} - \boldsymbol{s}^{\boldsymbol{r,e}}_{\boldsymbol{jt}}$

Remark 5. Bayer and Timmins (2007) suggest an alternative instrumental variables approach that also follows from the logic of a neighborhood choice model. In our notation, they suggest the use of observed amenities from neighborhood k as IV for $\boldsymbol{s_{jt}}$, for $k \neq j$. They exploit the idea that $\hat{v}^r_{jt}$ reflects the value of choice j as of period t, while $\boldsymbol{s_{jt}}$ reflects the *relative* value of choice j (relative to all choices $k \neq j$) as of period t. A different implementation of their idea suggested by our approach is to use $\hat{v}^r_{kt}$, $k \neq j$ as IV for $\boldsymbol{s_{jt}}$. This may be particularly useful when amenities are rarely observed (for instance when t varies at high frequency, as in our application.)

Remark 6. Recently, Bayer et al. (2016) have developed a feasible procedure to estimate a dynamic model of neighborhood choice that builds on well known methods in dynamic demand estimation (e.g., Hotz and Miller (1993) and Rust (1987)). We follow their estimation approach with two main departures. First, we are interested in identifying only the causal effects of expected endogenous amenities on households' cumulative utilities. In contrast, Bayer et al. (2016) are interested in identifying household *preferences*, which are typically understood as parameters of households' flow utilities. Hence, they need to make some assumptions in order to isolate the flow utility from the cumulative utility that are not necessary in our approach. In practice, this means that we can estimate the dynamic choice model imposing less structure on the state variables and on their expected transition over time. The second departure from Bayer et al. (2016) is that we do not explicitly allow for heterogeneity in terms of wealth. Despite its undeniable importance, particularly when studying the behavior of homeowners, allowing for such heterogeneity in our context would not only substantially increase the number of types of households, making the estimation infeasible (particularly for races other than Whites, as discussed in Bayer et al. (2016)), but would require additional unwarranted assumptions on expectations (see Remark 2). We do allow for different races to have on average different wealth, which is key to accommodate the relevant heterogeneity in wealth for a study of racial segregation.

5 Empirical Results

We present the results of our estimation and simulation procedures. In Table 2, we present estimates of the α^r parameter vectors from Equation (14) that represent the causal responses to endogenous amenities for each race. Each of the endogenous amenities is instrumented by the racial compositions of each neighborhood from periods $t-6$ to $t-12$, and the control variables v^r_{jt-1}, $r \in W, B, H, A$ are specified linearly.

Households of all races respond positively to neighborhoods with a greater share of residents of their own race, i.e., they are more likely to move into such neighborhoods. Such homophilic forces contribute to sorting patterns that in-

Table 2: Estimation Results - Responses to Endogenous Amenities

		White	Black	Hispanic	Asian
Responses to:	s_{jt}^B	-8.20***	14.08***	1.35***	-5.45***
		(0.34)	(0.38)	(0.27)	(0.34)
	s_{jt}^H	-9.96***	4.46***	28.38***	-2.87***
		(0.43)	(0.52)	(0.51)	(0.42)
	s_{jt}^A	-4.34***	-1.73***	-3.18***	17.18***
		(0.24)	(0.37)	(0.32)	(0.41)
R^2			0.49		
Num. Obs.			150,528		

Notes: All specifications include race-month fixed effects. Instrumental variables $(s_{jt-L}, \dots)$ are specified from periods $t-6$ to $t-12$, and control variables (v_{jt-k}^r) are specified linearly from period $t-1$. All standard errors clustered by race-month. *** - 99% significance.

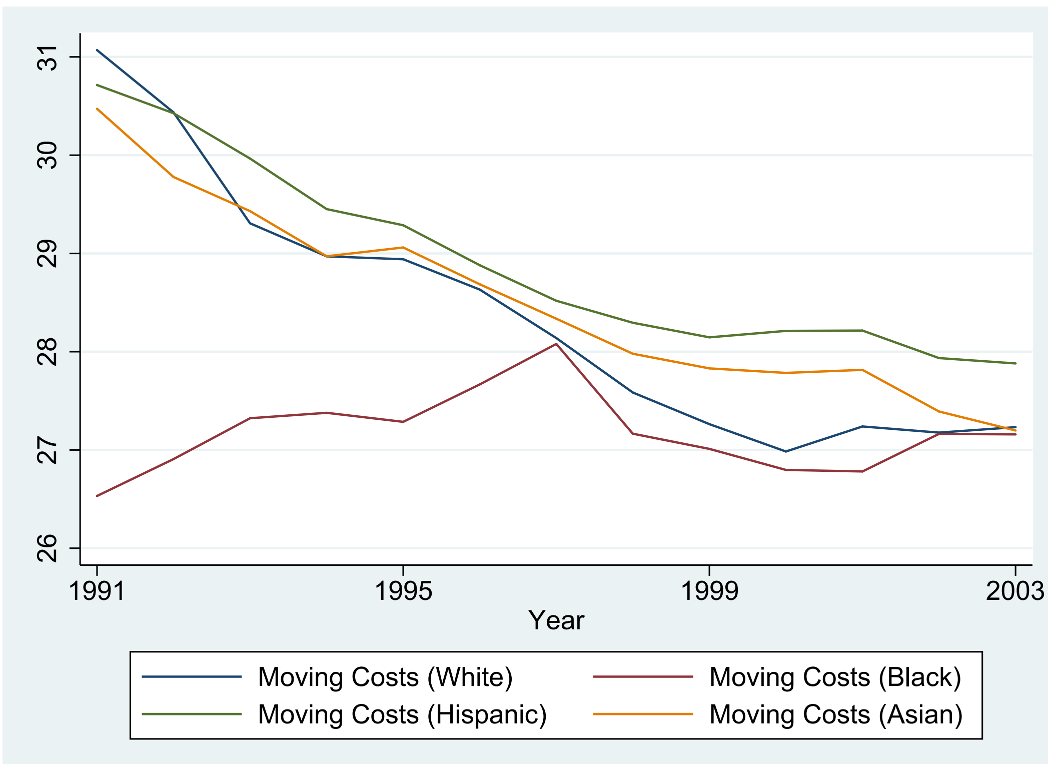
crease racial segregation. White and Asian households react negatively to greater shares of Black and Hispanic residents, whereas Black and Hispanic households react slightly positively to greater shares of Hispanic and Black residents respectively. Whites, Black and Hispanics all react slightly negatively to greater shares of Asian residents. These repelling forces can also contribute to sorting patterns that increase residential segregation.

In Figure 3, we present estimates of moving costs over time that vary by race and year.[13] White, Black and Asian households' moving costs are of similar magnitude, and they decrease moderately by about 10% over the period from 1991-2004 Hispanic households' have systematically lower moving costs that increase slightly over the first half of the sample period before following the same patterns as households of other races.

Incorporating these results into our simulation procedure paints a clear and interesting picture. For narrative purposes, we focus on the results of our simula-

[13]Note that the estimates of moving costs are denominated in utils, not dollars. Hence, they should only be interpreted in relation to each other. Confidence intervals for moving costs have been omitted for clarity, but each estimate of ϕ_t^r is statistically significantly different from zero at the 99% level.

Figure 3: Estimated Moving Costs Over Time



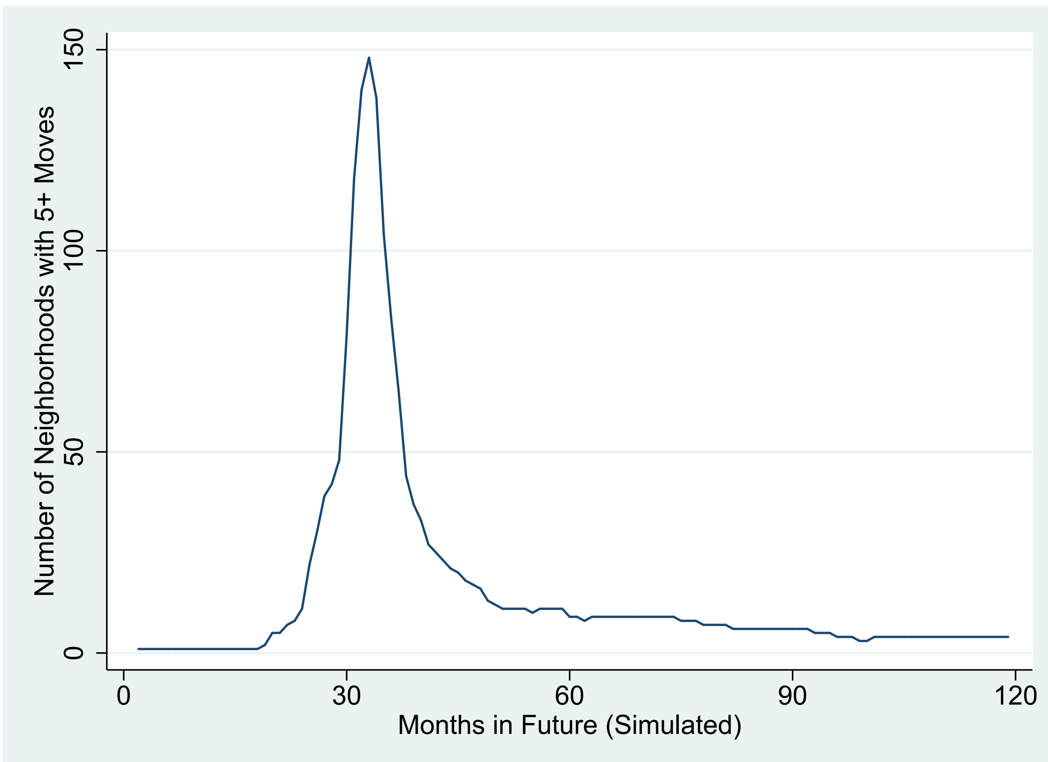
Notes: Race- and year- specific moving costs are estimated by GMM with moment conditions given in Equation (10). Moving costs denominated in units of utils.

tion from the final month of our sample, December 2004. In Figure 4, we present a graph of the number of neighborhoods that experience at 5 or more simulated net inflow of households of all races in a given month. We describe such neighborhoods as "in flux." Initially, very few neighborhoods are in flux, as relatively high moving costs generate substantial inertia in households' residential choices. Nevertheless, a small but increasing number of neighborhoods are in flux even early on. Over time, changes in these neighborhoods spill over to other neighborhoods, as their relative attractiveness changes as well. After approximately two years, the racial composition of enough neighborhoods have changed so that the relative attractiveness of neighborhoods in the Bay Area as a whole has changed enough in relation to moving costs. As a result, over two thirds of neighborhoods are found to be in flux at the peak of the simulation. After approximately four years, fewer than ten percent of neighborhoods remain in flux, as the majority of the Bay area has converged to a long run equilibrium state.

The outcome of this pattern of sorting is a change in the levels of segregation in the Bay Area, which we depict in Figure 5. In this figure, we present the average of the four-race Herfindahl index across all neighborhoods in the sample. A higher value means that the neighborhood is more dominated by household's of a single race, i.e., more segregated. Following the sorting pattern described above, aggregate segregation remains relatively unchanged until sorting accelerates after two years. After four years, aggregate segregation increases at a much lower rate. By the end of ten years, aggregate segregation in the Bay Area is simulated to be roughly 18% higher than what is observed initially.

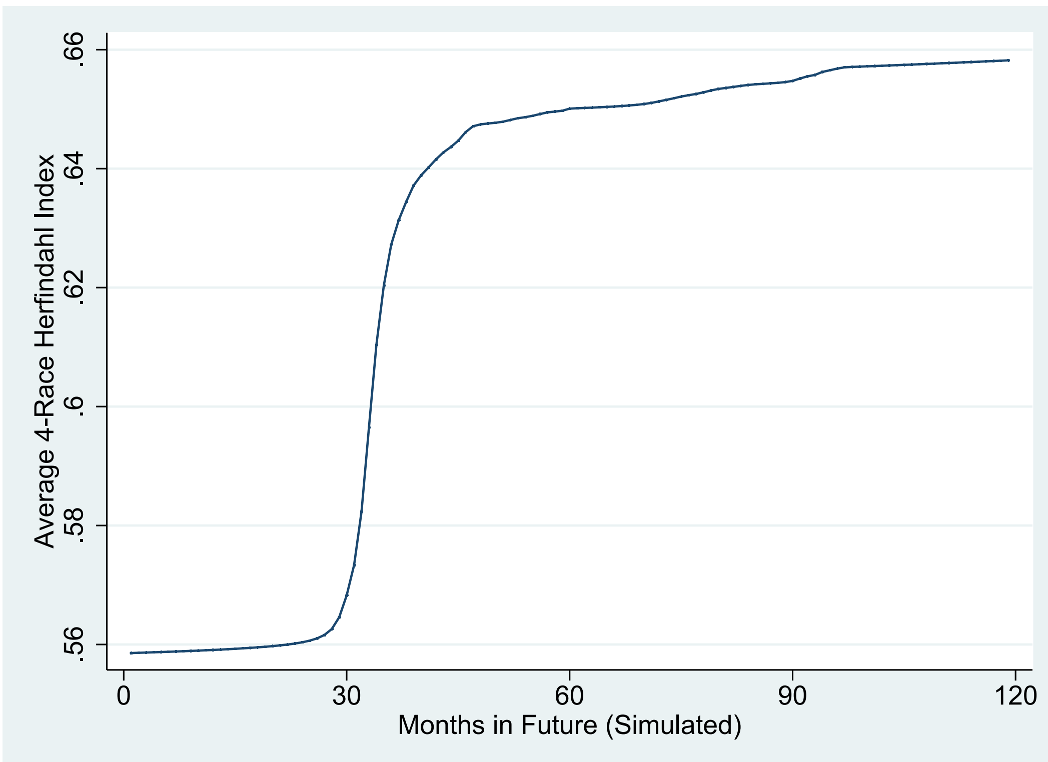
The aggregate increase in segregation is not experienced equally by households of different races. In Figure 6, we present the evolution of the Coefficient of variation of the shares of each race all neighborhoods. Intuitively, a higher value means that households of a given race are more concentrated in particular neighborhoods that feature, by construction, a greater fraction of same-race households. White households tend to be least concentrated (likely owing to their status as the most prevalent) , and over the course of the simulation end up slightly more diffusely spread over the Bay Area. Black, Hispanic and Asian households, however, end up more concentrated, with the coefficient of variation of the Hispanic share of neighborhoods increasing by a factor of six over the simulation period. This is

Figure 4: Number of Neighborhoods In Flux (Simulated)



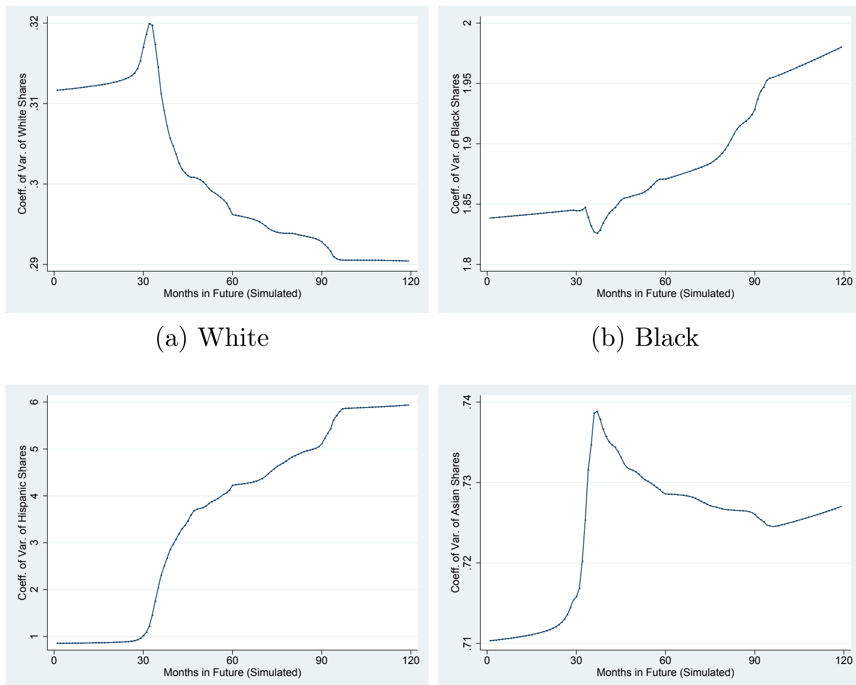
Notes: Figure shows the number of neighborhoods with five or more net inflows/outflows of all races (out of a total of 224 neighborhoods). Simulation begins in December 2004.

Figure 5: Trajectory of Aggregate Segregation (Simulated)



Notes: Figure shows the four-race Herfindahl Index of racial shares in each neighborhood averaged over the sample (i.e., $\frac{1}{J}\sum_j \sum_r \hat{s}^r_{ji}$ for the ith iteration). Simulation begins in December 2004.

Figure 6: Trajectories of Segregation Levels by Race (Simulated)



(a) White

(b) Black

(c) Hispanic

(d) Asian

Notes: Each panel shows the coefficient of variation of shares of a given race across all neighborhoods. Simulation begins in December 2004.

consistent with our large estimate of the coefficient on s^H_{jt} (see Table 2), which is more than twice as large as all other estimated racial response coefficients. The inherent tendency of Hispanic households to react very positively to highly Hispanic neighborhoods is a primary driver behind the aggregate increase in segregation that we find.

6 Conclusion

Any description of how households locate must appeal to the intuitive idea that neighborhoods constantly evolve: their amenities are not static and their residents are in flux. In this paper, we develop an empirical framework around this very notion that allows us to study how the aggregate phenomenon of segregation arises from the accumulation of many disaggregate residential choices that may be influenced, in part, by households' expectations of the racial compositions of neighborhoods. We find that households' choices are influenced by the racial compositions of prospective neighborhoods in systematic ways that are consistent with racial homophily. The result of this homophily is a tendency for segregation to increase in the absence of external shocks. Indeed, we find that at the end of our sample period, neighborhoods are out of equilibrium by roughly 20% in terms of racial segregation.

We view our approach as a platform for the empirical analysis of determinants of segregation that can be easily adapted to various contexts. For example, our approach can be used to study gentrification – neighborhood sorting between income groups – given appropriate data. In addition, our approach can be used to explore higher dimensional social interactions, as shown in our application for multiple racial groups (White, Black, Hispanic and Asian households). We hope that a deeper understanding of the determinants of sorting should enrich a growing literature on the manifold effects of segregation on a variety of important outcomes.

We should acknowledge several data deficiencies in our implementation that may affect the interpretation of our results. First, because accurate, high frequency data on the racial compositions of neighborhoods are not available from administrative sources, we must construct a data set that uses information from multiple sources. A key drawback of our approach is that we do not observe the racial composition of renters over time. This could be a particularly acute problem in a current analysis of urban segregation, as high home prices in many metropolitan areas (especially the San Francisco Bay Area) have driven increasing numbers of residents to the rental market. Because renters face relatively low moving costs, we would expect sorting to be even more pronounced than what we find. Second, because we do not observe the incomes and other attributes of households at high frequency, we cannot perform companion analyses of sorting along alternative dimensions. In addition to being of interest per se, a comparison of the degree of sorting along different socio-economic dimensions could prove valuable in revealing the importance of different cleavages in our society.

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A Instrumental Variables and the Choice Model

In this appendix, we demonstrate the explicit connection between our instrumental variables and our empirical model of residential choice. To see more formally how our identification strategy isolates the transitory component of interest, first note that the dynamic model of residential choice described above implies that $N^r_{jt} := N^r_{jt}(v^r_t, \phi^r_t, N^r_{jt-1}, I^r_t)$, that is, the total number of households of each race choosing neighborhood j in t can be written as a function of their current value of each neighborhood, $\boldsymbol{v^r_t}$, their current moving cost, ϕ^r_t, the total number of households of that race located in j as of $t-1$, and the total number of households of race r in t, I^r_t. Further, the endogenous variable of interest in equation (14) can be written as

$$s^r_{jt}(\boldsymbol{v_t}, \boldsymbol{\phi_t}, \boldsymbol{N_{jt-1}}, \boldsymbol{I_t}) = \frac{N^r_{jt}(v^r_t, \phi^r_t, N^r_{jt-1}, I^r_t)}{\sum_{r'} N^{r'}_{jt}(v^{r'}_t, \phi^{r'}_t, N^{r'}_{jt-1}, I^{r'}_t)} \tag{19}$$

where $\boldsymbol{v_t} := (\boldsymbol{v^W_t, v^B_t, v^H_t, v^A_t})$, $\boldsymbol{\phi_t} := (\phi^W_t, \phi^B_t, \phi^H_t, \phi^A_t)$, and $\boldsymbol{N_{jt-1}} := \boldsymbol{N_{jt-1}(v_t, \phi_t, N_{jt-2}}, \boldsymbol{I_t})$ and $\boldsymbol{I_t}$ are respectively the vectors including N^r_{jt-1} and I^r_t for all r. Re-writing equation (19) recursively:

$$\begin{aligned}
s^r_{jt}(\boldsymbol{v_t}, \boldsymbol{\phi_t}, \boldsymbol{N_{jt-1}}, \boldsymbol{I_t}) &= \frac{N^r_{jt}(v^r_t, \phi^r_t, N^r_{jt-1}, I^r_t)}{\sum_{r'} N^{r'}_{jt}(v^{r'}_t, \phi^{r'}_t, N^{r'}_{jt-1}, I^{r'}_t)} \\
&= \frac{N^r_{jt}(v^r_t, \phi^r_t, N^r_{jt-1}(v^r_{t-1}, \phi^r_{t-1}, N^r_{jt-2}, I^r_t)}{\sum_{r'} N^{r'}_{jt}(v^{r'}_t, \phi^{r'}_t, N^{r'}_{jt-1}(v^{r'}_{t-1}, \phi^{r'}_{t-1}, N^{r'}_{jt-2}, I^{r'}_t)} \\
&= s^r_{jt}(\boldsymbol{v_t}, \boldsymbol{v_{t-1}}, \boldsymbol{\phi_t}, \boldsymbol{\phi_{t-1}, N_{jt-2}}, \boldsymbol{I_t}).
\end{aligned} \tag{20}$$

Similarly, we can write $s^r_{jt-2}(\boldsymbol{v_{t-2}}, \boldsymbol{\phi_{t-2}}, \boldsymbol{N_{jt-3}}, \boldsymbol{I_{t-2}})$ recursively until period 0:

$$
\begin{aligned}
s^r_{jt-2}(\boldsymbol{v_{t-2}}, \boldsymbol{\phi_{t-2}}, \boldsymbol{N_{jt-3}}, \boldsymbol{I_{t-2}}) &= \frac{N^r_{jt-2}(v^r_{t-2}, \phi^r_{t-2}, N^r_{jt-3}, I^r_{t-2})}{\sum_{r'} N^{r'}_{jt-2}(v^{r'}_{t-2}, \phi^{r'}_{t-2}, N^{r'}_{jt-3}, I^{r'}_{t-2})} \\
&= \frac{N^r_{jt-2}(v^r_{t-2}, \phi^r_{t-2}, N^r_{jt-3}(v^r_{t-3}, \phi^r_{t-3}, N^r_{jt-4}, I^r_{t-2})}{\sum_{r'} N^{r'}_{jt-2}(v^{r'}_{t-2}, \phi^{r'}_{t-2}, N^{r'}_{jt-3}(v^{r'}_{t-3}, \phi^{r'}_{t-3}, N^{r'}_{jt-4}, I^{r'}_{t-2})} \\
&= ... \\
&:= s^r_{jt-2}(\boldsymbol{V_{t-2}}, \boldsymbol{\Phi_{t-2}}, \boldsymbol{N_{j0}}, \boldsymbol{I_{t-2}}), \qquad (21)
\end{aligned}
$$

where $\boldsymbol{V_{t-2}} := (\boldsymbol{v_{t-2}}, \boldsymbol{v_{t-3}}, \boldsymbol{v_{t-4}}, ..., \boldsymbol{v_0})$ $(\boldsymbol{\Phi_{t-2}} := (\boldsymbol{\phi_{t-2}}, \boldsymbol{\phi_{t-3}}, \boldsymbol{\phi_{t-4}}, ..., \boldsymbol{\phi_0}))$ denotes the three-dimensional (two-dimensional) matrix that comprises the full history of these matrices (vectors) up until $t-2$. Thus, we know that $s^r_{jt}(\boldsymbol{v_t}, \boldsymbol{v_{t-1}}, \boldsymbol{\phi_t}, \boldsymbol{\phi_{t-1}}, \boldsymbol{N_{jt-2}}, \boldsymbol{I_t})$ and $s^r_{jt-2}(\boldsymbol{V_{t-2}}, \boldsymbol{\Phi_{t-2}}, \boldsymbol{N_{j0}}, \boldsymbol{I_{t-2}})$ are correlated to each other across neighborhoods because of shocks in $\boldsymbol{V_{t-2}}$ (or initial condition $\boldsymbol{N_{j0}}$) that are correlated to either (a) $\boldsymbol{v_t}$ or $\boldsymbol{v_{t-1}}$ or (b) $\boldsymbol{N_{jt-2}}$. By controlling for $\boldsymbol{v_{jt-1}}$, we aim at ruling out the correlation between s^r_{jt} and s^r_{jt-2} due to channel (a), making sure this correlation only operates via channel (b).