# School Segregation and the Identification of Tipping Behavior: Supplementary Materials

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In this appendix, we discuss in detail three further realistic extensions of our framework which are not implemented in the paper for various reasons. Although these extensions have not been performed or even discussed in detail in the literature to the best of our knowledge, we believe that they may be necessary for a more complete analysis of tipping behavior.

## 1 Generalizing the Supply Side

In the main analysis, we assume that the supply of public schooling is perfectly elastic; that is, schools can instantaneously adjust their supply of school seats to accommodate any demand without affecting any other school amenities. Here, we formulate a more general version of the model outlined in the main text which includes prices, generalizing our simulation procedure to explicitly take into account endogenous adjustments in the supply of schooling. The implementation of this procedure would require simultaneous estimation of the supply of schooling, which is beyond the scope of this paper.

We begin by modifying the demand equation as

$$\log n_{jt}^{gr} = \beta^{gr} s_{jt-1} + \theta^{gr} P_{jt-1}^{g} + C_{jt-1}^{gr} + \epsilon_{jt}^{gr} \tag{1}$$

by adding $P_{jt-1}^{g}$, the implicit price of grade $g$ instruction at school $j$.[1]

Define $\Phi_{jt-1} = \left(s_{1t-1}\left(s\right), ..., s_{j-1,t-1}\left(s\right), s, s_{j+1,t-1}\left(s\right), ..., s_{Jt-1}\left(s\right)\right)$ as the counterfactual vector of minority shares of all schools when $s_j = s$. The elements of this vector may differ depending on the details of the counterfactual; for instance, if the counterfactual level of $s$ is achieved through a sorting of students into and out of school $k \neq j$ only, then we would have $s_{kt-1}(s) \neq s_{kt-1}$ and $s_{j't-1}(s) = s_{j't-1} \quad \forall j' \neq k, j$.

Given consistent estimates of $\beta^{gr}$ and $\theta^{gr}$, we generalize the simulation process to identify

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[1]This implicit price can, for instance, be proxied for by the average rent in the school attendance area that allows the parent to send the child to grade $g$ in school $j$. Prices may vary by grade because attendance areas may differ across grades.

tipping behavior as follows:[2] for each grade in each school $j$ in year $t$, we pick counterfactual values of $\Phi_{jt-1}$ and recalculate the endogenous vector of prices for *all* schools, $\tilde{P}^g_{t-1} \equiv P^g_{t-1}(\Phi_{jt-1}) = \left(P^g_{1t-1}(\Phi_{jt-1}), ..., P^g_{Jt-1}(\Phi_{jt-1})\right)$, that balances demand with supply for grade $g'$ and each school $j' = 1, \ldots, J$ according to the equilibrium conditions

$$
\begin{aligned}
N^{g'}_{j't}(\Phi_{jt-1}, \tilde{P}^{g'}_{t-1}) &= \tilde{n}^{g'W}_{j't}\left(\Phi_{jt-1}, \tilde{P}^{g'}_{t-1}\right) + \tilde{n}^{g'M}_{j't}\left(\Phi_{jt-1}, \tilde{P}^{g'}_{t-1}\right) && (2)\\
\tilde{n}^{g'r}_{j't}\left(\Phi_{jt-1}, \tilde{P}^{g'}_{t-1}\right) &= \frac{n^{g'r}_{j't}\left(\Phi_{jt-1}, \tilde{P}^{g'}_{t-1}\right)}{\sum_k n^{g'r}_{kt}\left(\Phi_{jt-1}, \tilde{P}^{g'}_{t-1}\right)} \cdot N^{g'r}_t && (3)\\
n^{g'r}_{j't}\left(\Phi_{jt-1}, \tilde{P}^{g'}_{t-1}\right) &= \exp\left(\log\left(n^{g'r}_{j't}\right) + \hat{\beta}^{g'r}\left(s_{j't-1}(s) - s_{j't-1}\right)\right. && (4)\\
&\quad \left. + \hat{\theta}^{g'r}\left(P^{g'}_{j't-1}(\Phi_{jt-1}) - P^{g'}_{j't-1}\right)\right)
\end{aligned}
$$

where $N^{g'r}_t = \sum_k n^{g'r}_{kt}$, $s_{jt-1}(s) \equiv s$, and $N^{g'}_{j't}\left(\Phi_{jt-1}, \tilde{P}^{g'}_{t-1}\right)$ is the grade $g'$ supply of school $j'$ in $t$, which would be estimated using appropriate techniques (e.g., Berry, Levinsohn and Pakes (1995)). Equation (2) equates supply and demand in each school-grade. Equation (3) rescales simulated demand for each school-grade, ensuring that we re-sort only those students who are actually observed in $t$. Equation (4) captures the fact that simulated enrollments are derived from the estimated demands for schooling, which are generally a function of prices as well. Given such an equilibrium, we can compute $S_j(\Phi_{jt-1})$ as

$$
S_j\left(\Phi_{jt-1}, \tilde{P}^K_{t-1}, \ldots, \tilde{P}^{12}_{t-1}\right) = \frac{\sum_g \tilde{n}^{gM}_{jt}\left(\Phi_{jt-1}, \tilde{P}^g_{t-1}\right)}{\sum_g \tilde{n}^{gM}_{jt}\left(\Phi_{jt-1}, \tilde{P}^g_{t-1}\right) + \tilde{n}^{gW}_{jt}\left(\Phi_{jt-1}, \tilde{P}^g_{t-1}\right)} \quad (5)
$$

[2]The assumption of perfectly elastic school supply in our baseline analysis only affects simulation of $S_{jt}$, *not* the demand estimation. To the extent that $P^g_{jt-1}$ *and* $P^g_{kt-1}$, $k \neq j$, are both affected by a change in $s_{jt-1}$, $\beta^{gr}$ captures the full reduced-form effect of a change in $s_{jt-1}$ on race $r$ demand. This includes both the direct effect of minority share on demand and any indirect effects due to concomitant changes in the prices of any school. During simulation, when $s \neq s_{jt-1}$, non-linear effects in price are less likely to be fully captured by the parameter estimates at $s_{jt-1}$, and this problem will be exacerbated for counterfactual values of $s$ far from $s_{jt-1}$. Estimating demand more flexibly as done in the main text can mitigate this issue, particularly with a rich data set with great heterogeneity in $s_j$ within schools and over time as in our case.

## 2 Individual Level Data

We conduct our empirical analysis using school level data as opposed to individual level data.[3] Our approach could be implemented with individual data allowing parents to have systematically heterogeneous preferences based on race, income, family education, etc. However, estimating this richer substitution pattern involves an important trade-off, as doing so may imply a less useful counterfactual in the simulation exercise.

To illustrate, assume that we observe parental income, and rich, White (and minority) parents have a different preference for the minority share than poor, White (and minority) parents. In this case, controlling for the income of the household in the first stage when estimating $\beta^{gr}$ may generate a less useful counterfactual in the simulation stage than not controlling for it. By not controlling for income, we do not hold income constant for each counterfactual value of $s$, so the simulation should be interpreted as allowing the *average* (in all other characteristics) White or minority student to flow in or out of the school. If Whites tend to be richer than minorities, then the simulation performed in the paper involves a flow of White (and/or minority) students and implicitly allows parents to re-sort using the statistical information that Whites tend to be richer than minorities.

If instead we hold income constant in the simulation, then we would necessarily be considering flows of White and minority students of equal levels of income. And if we control for more demographic characteristics, such a simulation may not even constitute a feasible reallocation of parents, as there may not be parents of different races and with the same level of all other observable characteristics. Thus, by not controlling for non-racial characteristics of parents when estimating demand (even if individual data was observable), we can conduct a more relevant simulation procedure. Of course, race should then be interpreted as a proxy for the whole bundle of characteristics of a typical student of that race, which is what we explicitly do. This interpretation is in line with the empirical literature on school segregation (e.g., Jackson (2009) and Billings, Deming and Rockoff (2012)).

More generally, we could also use individual level data to add more social amenities to the analysis. Consider for instance the case where we observe demand stratified by race-income. That would allow us to identify $\delta_{jt}^{gr'}$, where

$$r' \in \{\text{Rich-White,Poor-White,Rich-Minority,Poor-Minority}\}$$

We would then be able to analyze tipping behavior with respect to three social amenities rather than just one, where the four groups would be allowed to have heterogeneous preferences over these three amenities (as well as the private amenities). We are unable to perform this analysis for lack of data,

[3] Individual level student data in the state of California is unavailable in restricted or unrestricted formats. The California Department of Education mentions that "California Standardized Testing and Reporting (STAR) Program test results for schools, counties, districts, and the state are available at this site. (...) Important Note: Test results for individual students are available only to parents/guardians and may be obtained only from the schools and school districts where students were tested. Individual student results are not available on the Internet nor from the California Department of Education." Source: http://star.cde.ca.gov/star2012/index.aspx.

but we do offer an extension of our framework with two social amenities to illustrate the complex analysis that can be performed with our approach. Although stratifying demand by race-income is likely to provide some interesting insights into tipping behavior, it does come with at the same cost related to the plausibility of the counterfactual.

## 3 Forward Looking Behavior

A fundamental assumption of the Schelling model and its successors is the assumption of myopia – i.e., agents do not engage in forward looking behavior. Following this literature, we make this assumption throughout the paper. If however people are forward looking, our estimates are not likely to change.

To see this, let $\mathbb{E}[s_{jt}]$ be the expected minority share in school $j$ just before parents make their decision in $t$. If parents are forward looking, then we can express the error in our specification as $ME_{jt} \equiv \mathbb{E}[s_{jt}] - s_{jt-1} \neq 0$. We can then rewrite the demand equation as

$$\log n_{jt}^{gr} = \beta^{gr} s_{jt-1} + C_{jt-1}^{gr} + \tilde{\epsilon}_{jt}^{gr}$$

where $\tilde{\epsilon}_{jt}^{gr} = \epsilon_{jt}^{gr} + \beta^{gr} ME_{jt}$ and $\epsilon_{jt}^{gr}$ is the regression error with myopia. First, note that only the component of $\beta^{gr} ME_{jt}$ that varies across schools within neighborhood for a given grade, race and year will not be absorbed by the fixed effects. Second, the remaining component has to be (1) correlated to both the enrollments in $t-2$ of the IV cohort and the enrollments in $t$ of some cohort that attends the school, and (2) uncorrelated to the enrollments in $t-1$ of any of the control cohorts. Hence, the argument for the validity of the IV is exactly the same as before. Specifically, note that our control variables, $\log n_{jt-1}^{gr}$, would reflect parents' expectations of the levels of future amenities, with expectations formed as of $t-1$. Thus, our IV would be invalid only if parents made their enrollment decision in $t-2$ using some information about future amenities that was not relevant when their child enrolled in $t-1$ but became relevant again at the time of decisions taken in $t$.

## References


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